Electromagnetics in geosciences - Understanding physical mechanisms

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Abstract

Electromagnetic (EM) methods could seem somehow fuzzy to Exploration and Production Geoscientists working in the field of seismic methods, relying on propagation of elastic waves. That article aims to facilitate understanding of EM methods. Their definition, possibilities, limits and applications in Geosciences. Explanations shall focus on physics keeping as a motto the will to pass some hints to put readers on track and let them identify clearly the most common physical mechanisms at stake for different EM methods available in the Industry: inductive, galvanic and geometric effects. Potential, DC and induced polarization methods are not reviewed.

History and Purposes

Beginning with History and Purposes was quite mandatory to understand why and how electromagnetism was considered to explore the subsurface when dealing with geoscience matters.

On the historical side, Electrical methods dedicated to Geosciences began with the Schlumberger Brothers in the late 20's (Figure 1). In a century, technology started from a simple DC resistivity method and went up to complex Electromagnetic methods implying natural or controlled sources, from 1D to 4D models, from acquisition in boreholes to land, air and sea. EM is now used in, tectonic studies, Oil & Gas and Mining Industry, Geothermal and Near surface studies (civil engineering, groundwater monitoring and environmental purposes). First known success occurred with mineral exploration on highly conductive sulfide metal ores bodies.

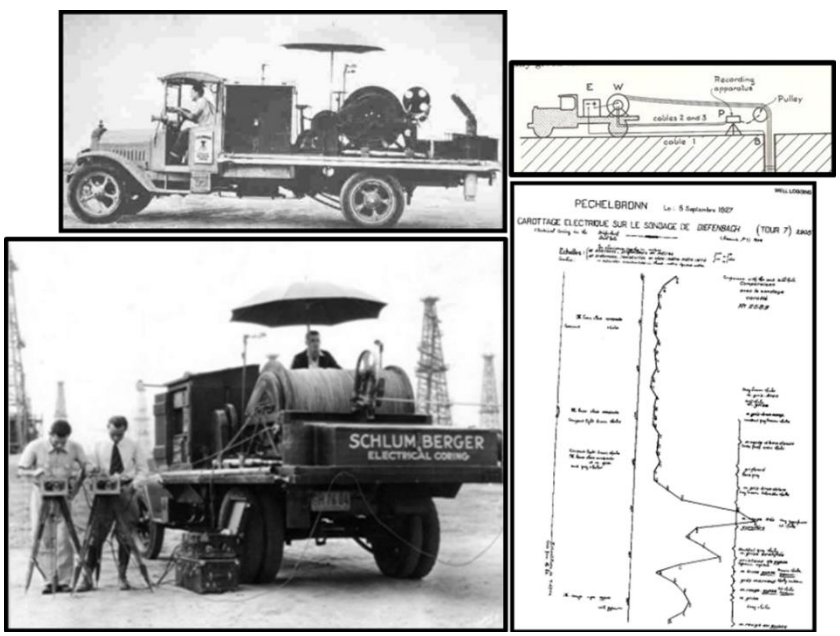

Figure 1: September 5, 1927 - A technology that will revolutionize oil and gas exploration – an electric downhole well log – is first applied in Pechelbronn, France.

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A behavior of an EM field is controlled by 3 main parameters:

- Electrical conductivity/resistivity
- Dielectric permittivity
- Magnetic susceptibility

The Electrical conductivity or resistivity is the most important for DC and low frequency methods (i.e. below 1 kHz for MT, AMT, mCSEM, AEM) whereas the Electric permittivity is the most important for high frequency methods (i.e. above 1 MHz for GPR).

Predominance of the electrical resistivity and the very large domains of application could be explained considering simple physical phenomena:

- Electrical resistivity decreases with the rock water content,
- Electrical resistivity increases with the rock hydrocarbon (HC) content,
- Electrical resistivity decreases with the rock temperature,
- …

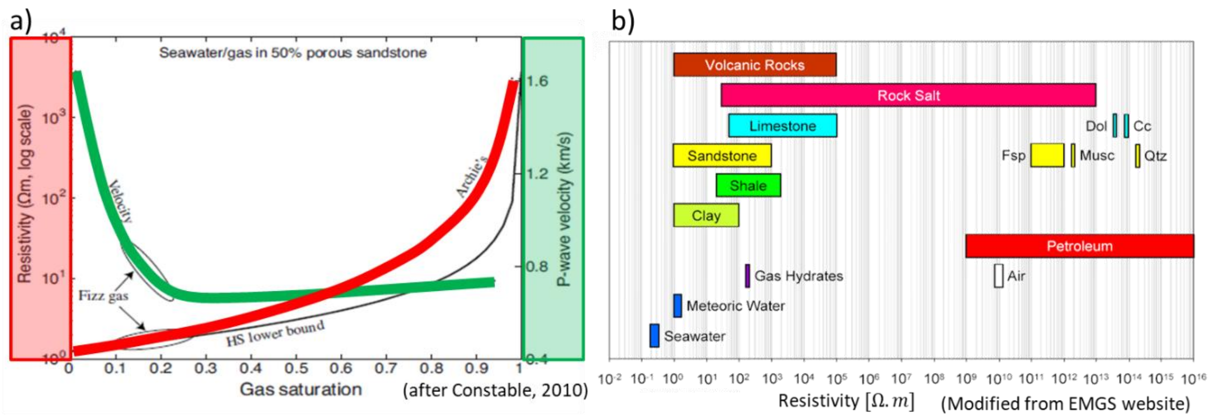


Figure 2: a) effects of a change of gas saturation on resistivity and acoustic waves after Constable (2010), b) materials resistivity range after EMGS website.

A fair digression about Oil & Gas sector should be made to explain enthusiasm occurring during the two last decades for EM methods. Figure 2 made such an interest obvious. Figure 2.a, after Constable (2010), shows the effects of a change of gas saturation in the pore fluid of a 50% porous sandstone on compressional wave velocity and electric resistivity. Implicitly that is a way to compare seismic and EM methods benefits. Two curves could be observed: The green curve exhibits effect of gas on compressional wave velocity while the red shows the effect of gas on the electric resistivity. The largest effect on acoustic velocity occurs for the very first few percent of gas fraction, but on the contrary fizz gas has little effect on electric resistivity, which does not increase significantly until gas saturations reaches 70% - 80%. That means we could fairly expect EM methods to assess commercial interest of a gas bearing reservoir where seismic methods could fail. The chart in Figure 2.b, shows materials which are part of E&P investigations and are distributed over a massive range of orders of magnitudes regarding the electric resistivity. Resistivity variations in sediments are controlled by variations of porosity, permeability, pore connectivity geometry and the fluids contained by the pores. As standard approximations, the Industry often takes 0.3 $\Omega.m$ for seawater, 1.5 to 3 $\Omega.m$ for sediments saturated with brine and up to 100 $\Omega.m$ for hydrocarbon bearing reservoirs. Almost two orders of magnitude between sediments containing brine and those containing HC. On the other side, elastic waves could not even pretend to 1 order of magnitude of difference.

Mathematical Notions

Before tackling EM theory, some commonly used mathematical operators are reminded in Figure 3 standing as a tool gathering. These are illustrated with simple examples to access their physical meaning at a glance.

Physical meaning of some linear operators helps with EM theory understanding:

- The **gradient** points in the direction of maximum increase of the function:

$$\overrightarrow{\mathrm{grad}}\ \mathrm{F} = \vec{\nabla}\mathrm{F} = \left(\frac{\partial \mathrm{F}}{\partial x}, \frac{\partial \mathrm{F}}{\partial y}, \frac{\partial \mathrm{F}}{\partial z}\right)$$

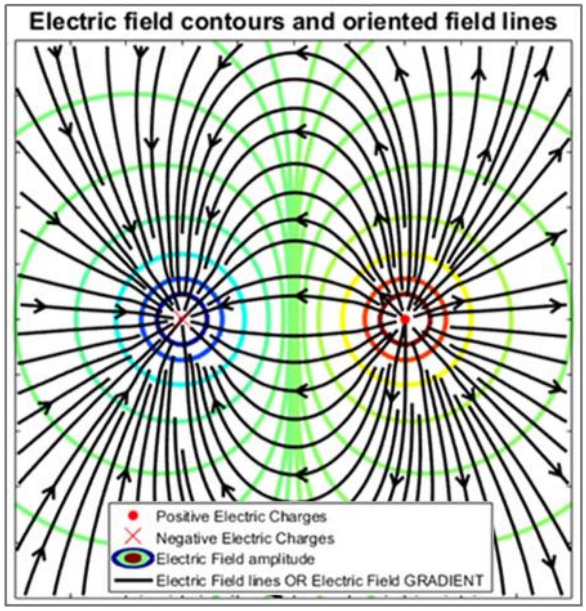
- The **divergence** is a measure of how much the vector function spreads out from the point in question:

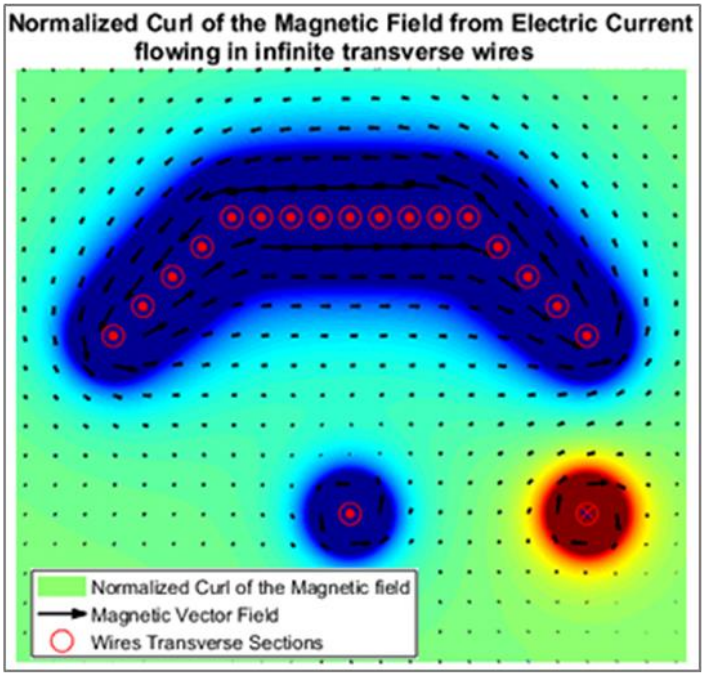
$$\mathrm{div}\ \vec{\mathrm{F}} = \vec{\nabla}.\vec{\mathrm{F}} = \frac{\partial \mathrm{F_x}}{\partial x} + \frac{\partial \mathrm{F_y}}{\partial y} + \frac{\partial \mathrm{F_z}}{\partial z}$$

- The **curl** is a measure of how much the vector field "curls around" the point in question.

$$\overrightarrow{\mathrm{curl}}\ \vec{\mathrm{F}} = \vec{\nabla} \times \vec{\mathrm{F}} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (\mathrm{F_x}, \mathrm{F_y}, \mathrm{F_z}) = \left(\frac{\partial \mathrm{F_z}}{\partial y} - \frac{\partial \mathrm{F_y}}{\partial z}, \frac{\partial \mathrm{F_x}}{\partial z} - \frac{\partial \mathrm{F_z}}{\partial x}, \frac{\partial \mathrm{F_y}}{\partial x} - \frac{\partial \mathrm{F_x}}{\partial y}\right)$$

With $\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ and $\vec{\mathrm{F}} = (\mathrm{F_x}, \mathrm{F_y}, \mathrm{F_z})$.





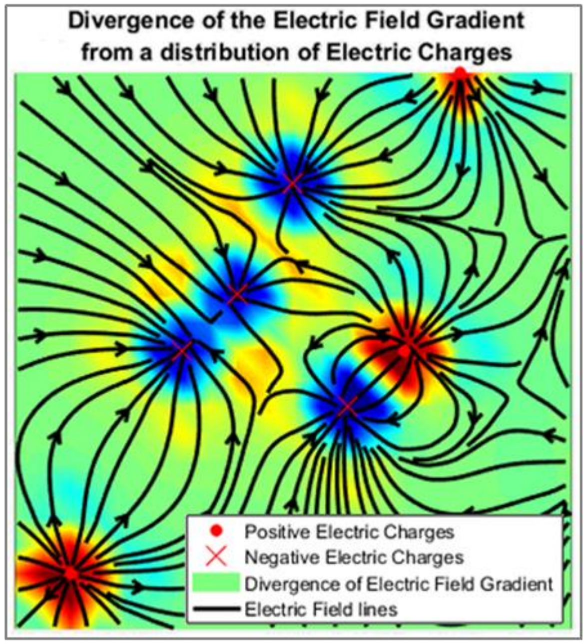


Figure 3: Mathematical tools definitions.

$$\mathrm{div}\left(\overrightarrow{\mathrm{curl}}\vec{\mathrm{F}}\right) = 0 \quad [1]$$

$$\overrightarrow{\mathrm{curl}}\left(\overrightarrow{\mathrm{grad}}\ \mathrm{F}\right) = \vec{0} \quad [2]$$

$$\overrightarrow{\mathrm{curl}}\ \left(\overrightarrow{\mathrm{curl}}\vec{\mathrm{F}}\right) = \overrightarrow{\mathrm{grad}}\ \left(\mathrm{div}\ \vec{\mathrm{F}}\right) - \mathrm{div}\ \left(\overrightarrow{\mathrm{grad}}\ \mathrm{F}\right) \quad [3]$$

$$= \overrightarrow{\mathrm{grad}}\ \left(\mathrm{div}\ \vec{\mathrm{F}}\right) - \Delta\vec{\mathrm{F}} \text{ or } \overrightarrow{\mathrm{grad}}\ \left(\mathrm{div}\ \vec{\mathrm{F}}\right) - \nabla^2\vec{\mathrm{F}}$$

Green-Ostrogradsky Theorem: $\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \oiint_S \vec{F} \cdot \vec{dS}$ [4]

Stokes Theorem: $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \cdot \vec{dS} = \oint_C \vec{F}.\vec{dl}$ [5]

EM theory

Maxwell's equations

Handling Maxwell's equations (Eq. [6] to [9]) relying on such operators alongside with handy identities and theorems (Eq. [1] to [5]) shall explicit how EM waves travel in the subsurface and what are the phenomena at stake.

But a bit of History first. In the 19th century, Clerk Maxwell (1831-1879), Scottish physicist, gathered Electric and Magnetic Theory to create EM (with the help of Faraday). He developed the notion of EM planar wave with an electric field orthogonal to the magnetic field. Maxwell's theory is based on 4 formulae and first of all, no sources in our volume of interest. They are considered sufficiently faraway, which corresponds to a valid hypothesis to explain how MT methods work for example.

Quickly, Gauss's equations (Eq. [6] to [7]) give information about fields lines geometry while the others (Eq. [8] to [9]) link time and space, and in the end, the gathering of equations enables to describe EM state everywhere at all times.

Then, conservation of the charge brings $\nabla.\vec{j} = 0$. It comes first from divergence of Maxwell Ampère. Same as material balance in mechanics. Variation in time of the electric charge density is zero since free charges get back to a bounded state very fast, in our context, with geological materials, which are often not very conductive.

Maxwell's equations could be found expressed with $\vec{b}$ and $\vec{d}$ (magnetic flux density and electric displacement). They are respectively linked to $\vec{h}$ and $\vec{e}$ with magnetic and electric polarization of the matter; which are in their turn respectively linked to magnetic permeability and dielectric permittivity.

$$\text{Gauss: } \vec{\nabla}.\vec{h} = 0 \quad [6]$$

$$\text{Gauss: } \vec{\nabla}.\vec{e} = \frac{\rho}{\varepsilon} \quad [7]$$

$$\text{Faraday: } \vec{\nabla} \times \vec{e} = -\mu\frac{\partial\vec{h}}{\partial t} \quad [8]$$

$$\text{Maxwell} - \text{Ampère: } \vec{\nabla} \times \vec{h} = \varepsilon\frac{\partial\vec{e}}{\partial t} + \vec{j} \quad [9]$$

where $\vec{e}$ is the electric field [V/m], $\vec{h}$ the magnetic field (A/m), $\vec{j}$ the current density (A/m²) and ρ electric charge density [C/m^3].

For each formula in the article, lower case fonts are used for time domain while upper case fonts are used for The constitutive relationships bounding the fields, charges and currents are frequency-dependent as:

- $\vec{D} = \varepsilon(\omega, \ldots)\vec{E}$
- $\vec{B} = \mu(\omega, \ldots)\vec{H}$
- $\vec{J} = \sigma(\omega, \ldots)\vec{E}$

The tensors ε, μ and σ are the dielectric permittivity, the magnetic permeability and the electric conductivity.

Realistic hypothesis could still be taken when dealing with geosciences:

- Relative magnetic permeability is equal to Magnetic permeability in a vacuum, i.e. $\mu = \mu_0 = 4\pi.\ 10^{-7}$ H/m.
- ε and σ are real number.
- ε and σ are not dependent on the frequency.

Bringing in time domain:

$$\vec{d} = \varepsilon\vec{e}$$

$$\vec{b} = \mu\vec{h}$$

Ohm' s law, the "heart" of the galvanic effect, is then:

$$\vec{j} = \sigma\vec{e} \qquad [10]$$

Ohm's law and Maxwell's equations represent the "basement" for exploration techniques described in that article. Understanding other milestones of EM methods such as skin depth or importance of dielectric permittivity may require getting back to some more physics exploring the so-called Maxwell's equations.

Inductive effect

When a target is more conductive than the surrounding formations, inductive currents lines - induced by an EM wave - draw circles restrained to the conductive target. That leads to the creation of a magnetic dipole of which

Fourier domain.

the orientation depends on the target geometry. That is what is called inductive effect, on what rely many EM methods with time-variant electromagnetic fields. Figure 4 illustrates that phenomenon, fairly described by the Maxwell's equations.

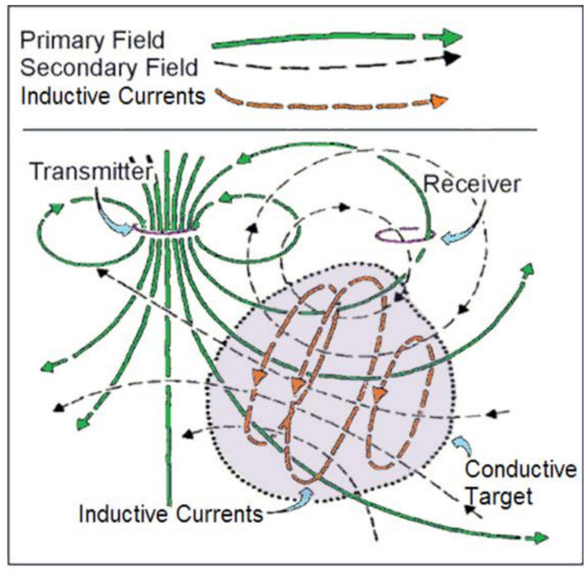


Figure 4: Sketch describing the inductive effect.

Maxwell's equations could then be investigated to have a better physical insight in what induction corresponds to if we handle div and $curl$ operators within Maxwell's equations, it comes in the Fourier domain:

$$\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} - i\omega\mu\sigma\vec{E} = \vec{0} \qquad [11]$$

The upper-cased delta symbol refers to Laplacian operator defined in eq. [3].

Then, if k is defined such as $k^2 = \omega^2\mu\varepsilon - i\omega\mu\sigma$, eq. [11] becomes the Helmoltz equation and k is called the wave number as follows:

$$\Delta\vec{E} + k^2\vec{E} = \vec{0} \qquad [12]$$

Here, expression of the Helmoltz equation is arbitrary expressed with the electric field $\boldsymbol{E}$. The same equation could be found with the magnetic field from the Maxwell's equations.

Dealing with geological matters and low frequencies, generally below 1 kHz (depending on the medium at stake and environment of acquisition), the expression of the wave number could be simplified ($\omega^2\mu\varepsilon \ll \omega\mu\sigma$) and back in time, we could recognize the so-called diffusion equation, commonly known as the heat equation, eq. [13].

$$\omega^2\mu\varepsilon \ll \omega\mu\sigma, \text{then } k \approx \sqrt{-i\omega\mu\sigma} \quad [13]$$
$$\text{and } \Delta\vec{e} - \mu\sigma\frac{\partial\vec{e}}{\partial t} = \vec{0}$$

The solution in 1D is as follows for progressive part: $\vec{E} = \vec{E}_0^+ e^{i(\omega t - kz)}$

k has then a real and an imaginary part. Split in the solution, a negative exponential term occurs with a purely real factor corresponding to an amplitude attenuation term. Skin depth is extensively used in the literature, here and commonly quoted as δ with $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \approx 503\sqrt{\rho/f}$ and $\rho = 1/\sigma$ to quantify the amplitude loss of the diffusing wave. The more the conductivity and the frequency are high, the more the signal gets attenuated. If the wave travels of a distance equals to delta, amplitude loss reaches 67%.

A similar effect could be observed on the phase with occurrence of a delay since phase velocity shall now depend on the frequency and the conductivity.

That described effect on amplitude and phase of a travelling wave is also called the inductive effect.

Propagating waves

In case of high frequency method ($\omega^2\mu\varepsilon \gg \omega\mu\sigma$), k could be approximated as $\sqrt{\omega^2\mu\varepsilon}$ leading to $\Delta\vec{e} - \mu\varepsilon\frac{\partial^2\vec{e}}{\partial t^2} = \vec{0}$ in time domain.

The solution in 1D is as follows: $\vec{e_0}cos(\omega t - \vec{k}.\vec{r})$. The amplitude and phase of the travelling wave are unchanged in that particular case. However, practically, attenuation always exists and could not be neglected for methods relying on EM wave propagation such as GPR.

Boundary conditions - Galvanic effect and Snell-Descartes' law

Defining boundary conditions starts from the 4 Maxwell's equations, the Green-Ostrogradski Theorem (eq. [4]) and the Stokes Theorem (eq. [5]). If elementary loop and volume are taken over an interface between two media with different EM properties quoted with indices 1 and 2, that comes:

1 – From Conservation of the charge:

$\left(\vec{j_1} - \vec{j_2}\right).\vec{n} = 0$ with $\vec{n}$ the unit vector normal at the interface, Ohm's law brings normal component of the electric field is not continuous from medium 1 to medium 2. That corresponds to the origin of the galvanic effect.

In a general way, when a target is more conductive than the surrounding formations, currents (induced by the EM wave) are focused within the target. That channeling effect is the consequence of a charge accumulation at the surface of the target which creates an electric depolarization field in opposition of the primary electric field as shown in Figure 5.

2 – Snell-Descartes: $k_1\ sin(\theta_1) = \ k_2\ sin(\theta_2)$.

As widely known with seismic methods, a critical angle could then be observed for diffusing and propagating EM waves.

Geometric amplitude decay

When the EM wave is generated by a local source, that could not be considered as planar and a geometric decay is observed.

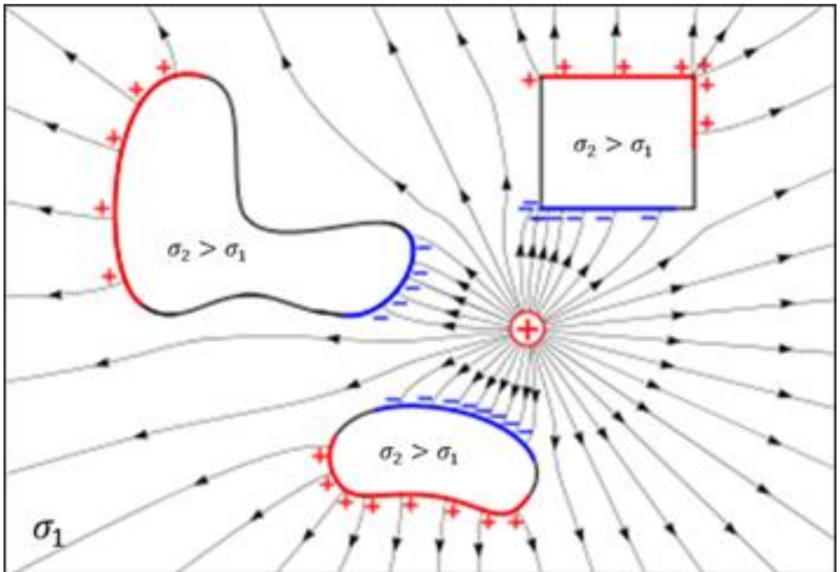


Figure 5: Illustration of the galvanic effect.

Bringing an EM source slightly changes the time-variant Maxwell's equations. In case an electric source is considered, Maxwell-Ampère is turned to:

$$\text{Maxwell} - \text{Ampère: } \vec{\nabla} \times \vec{h} - \varepsilon \frac{\partial \vec{e}}{\partial t} - \vec{j} = \vec{J_e^s} \quad [14]$$

With $\vec{J_e^s}$ the current source density from an electric source. Maxwell-Faraday and Gauss formulae remain unchanged. Solving such a new system of equations requires use of a mathematical trick, namely "vector potentials" through the following workflow:

1. Use of vector potential ($\vec{H_e} \equiv \vec{\nabla} \times \vec{A}$).

2. Getting back to the Helmoltz equation with $\vec{A}$:

$$\Delta\vec{A} + k^2\vec{A} = -\vec{J_e^s}$$

3. Resolution of the Helmholtz equation, shall bring:

$$\begin{cases} E_x = \frac{IL}{4\pi\sigma r^3}\left[\frac{3x^2}{r^2} - 1\right] \\ E_y = \frac{IL}{4\pi\sigma r^3}\left[\frac{3xy}{r^2}\right] \\ E_z = \frac{IL}{4\pi\sigma r^3}\left[\frac{3xz}{r^2}\right] \end{cases}$$

Vector potentials are employed to solve the new system of Maxwell's equations with the assumption the magnetic field is derived from a purely rotating field, the vector potential *A*. That subsequently enables to get back to the so-called Helmoltz equation using the vector potential with known solution. And it brings a decay of the electric field varying like the inverse of the cubic range, with IL the moment of the dipole (*I* the current intensity and *L* the distance between the two electrodes).

Conclusions

Figure 6 is a résumé of different phenomena introduced in the article, after Constable (2010).

Geometric: Geoscientists dealing with DC methods are quite familiar with that phenomenon with a (1/ cubic *range*) in the low-frequency limit.

Galvanic: Increased by the contrast of conductivity and occurrence of normal components of electric currents at the interface between 2 media.

Inductive: Responsible for amplitude decay and phase delay function of the frequency and conductivity.

Difficulty in understanding EM methods is that several mechanisms are at work to produce changes in amplitude and phase. The first is geometric spreading from the transmitter. The second is the galvanic effect associated with current passing across a conductivity boundary. Like the geometric effects, it has no effect on signal phase. Finally, the process of inductive attenuation and phase shift occurs when the skin depths are comparable to the distance over which the EM energy has traveled (Constable, 2010).

Chargeability has been deliberately put aside. However, that phenomenon could be investigate when dealing with minerals exploration and induced polarization techniques.

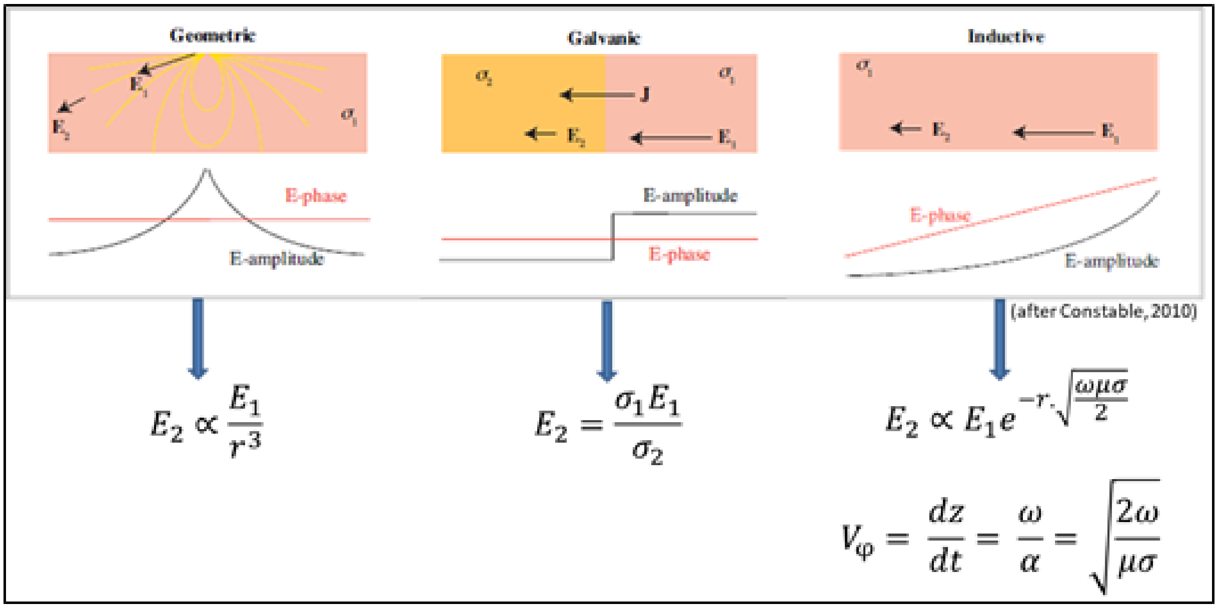


Figure 6: Physical phenomena résumé.

Acknowledgements

We thank IFPEN for publishing that article destined to help beginners to tackle EM methods and underlying theory.

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Glossary

Methods Acronyms:
EM = ElectroMagnetics
MT = MagnetoTellurics
AMT = Acoustic MagnetoTellurics
AEM = Airborne ElectroMagnetics
mCSEM = marine Controlled Source EM
GPR = Ground Penetrating Radar
Physical quantities:
ρ = electrical resistivity measured in Ω.m
σ = electrical conductivity measured in S/m
$\vec{j}$ = Current density measured in A/m2

$\vec{e}$ = Electric field strength measured in V/m
$\vec{h}$ = Magnetic field strength measured in A/m
$\vec{b}$ = Magnetic Flux Density measured in Tesla
$\vec{d}$ = Electric displacement measured in Coulombs/m2
ε_0 = Electric permittivity of a vacuum (F/m); $\varepsilon_0 \approx 8.854.10^{-12}$ F/m
μ_0= Magnetic permeability of a vacuum (H/m); $\mu_0 = 4\pi.\ 10^{-7}$ H/m
ε = Electric permittivity of some matter (F/m)
μ = Magnetic permeability of some matter(H/m)
$\vec{\nabla} = Gradient, \vec{\nabla}. = Divergence, \vec{\nabla}\times = Curl, \Delta = Laplacian$
$\omega(= 2\pi f)$ angular frequency in rad/s