Electromagnetics in geosciences – Theory to perform magneto-tellurics

Jean-François Girard

Unistra – EOST/IPGS – Université de Strasbourg, France.

Email : jf.girard@unistra.fr

Jean-Luc Mari

IFP Energies nouvelles, Rueil-Malmaison, France.

Email: jean-luc.mari@ifpen.fr

Abstract

The Magneto-telluric method is a geophysical exploration technic used for decades. It relies on the physical laws of plane wave theory. Used at low frequencies, it allows to image from several tens of meters to the deeper crust and further. But, as a particular case of the general theory of electromagnetism, it requires certain assumptions to be fulfilled. This short paper aims at presenting some important theoretical aspects assumed when using this method.

From Maxwell's equations to EM plane wave sounding

Let starting by the formulation of Maxwell equations in 3D, in a cartesian coordinates system (x,y,z).

They deal with 6 coupled differential equations with 6 unknowns

$$\frac{\partial \vec{h}}{\partial t} = -\frac{1}{\mu}\vec{\nabla}\times\vec{e} \qquad \frac{\partial \vec{e}}{\partial t} = \frac{1}{\varepsilon}\vec{\nabla}\times\vec{h} - \frac{1}{\varepsilon}\vec{j}$$

$$\begin{cases} \dfrac{\partial h_x}{\partial t} = \dfrac{1}{\mu}\left(\dfrac{\partial e_y}{\partial z} - \dfrac{\partial e_z}{\partial y}\right) \\ \dfrac{\partial e_y}{\partial t} = \dfrac{1}{\varepsilon}\left(\dfrac{\partial h_x}{\partial z} - \dfrac{\partial h_z}{\partial x} - j_y\right) \\ \dfrac{\partial h_z}{\partial t} = \dfrac{1}{\mu}\left(\dfrac{\partial e_x}{\partial y} - \dfrac{\partial e_y}{\partial x}\right) \end{cases} \qquad \begin{cases} \dfrac{\partial e_x}{\partial t} = \dfrac{1}{\varepsilon}\left(\dfrac{\partial h_z}{\partial y} - \dfrac{\partial h_y}{\partial z} - j_x\right) \\ \dfrac{\partial h_y}{\partial t} = \dfrac{1}{\mu}\left(\dfrac{\partial e_z}{\partial x} - \dfrac{\partial e_x}{\partial z}\right) \\ \dfrac{\partial e_z}{\partial t} = \dfrac{1}{\varepsilon}\left(\dfrac{\partial h_y}{\partial x} - \dfrac{\partial h_x}{\partial y} - j_z\right) \end{cases}$$

Figure 1 – Maxwell's equations in Cartesian coordinates

Expressions of Maxwell equations in 2D

If there is no variation along y, we are in a 2D case. The derivations along y vanish. We obtain two independent systems: H_x, H_z, E_y one side, and E_x, E_z, H_y on the other side.

$$\begin{cases} \dfrac{\partial h_x}{\partial t} = \dfrac{1}{\mu}\left(\dfrac{\partial e_y}{\partial z} - \cancel{\dfrac{\partial e_z}{\partial y}}\right) \\ \dfrac{\partial e_y}{\partial t} = \dfrac{1}{\varepsilon}\left(\dfrac{\partial h_x}{\partial z} - \dfrac{\partial h_z}{\partial x} - j_y\right) \\ \dfrac{\partial h_z}{\partial t} = \dfrac{1}{\mu}\left(\cancel{\dfrac{\partial e_x}{\partial y}} - \dfrac{\partial e_y}{\partial x}\right) \end{cases} \qquad \begin{cases} \dfrac{\partial e_x}{\partial t} = \dfrac{1}{\varepsilon}\left(\cancel{\dfrac{\partial h_z}{\partial y}} - \dfrac{\partial h_y}{\partial z} - j_x\right) \\ \dfrac{\partial h_y}{\partial t} = \dfrac{1}{\mu}\left(\dfrac{\partial e_z}{\partial x} - \dfrac{\partial e_x}{\partial z}\right) \\ \dfrac{\partial e_z}{\partial t} = \dfrac{1}{\varepsilon}\left(\dfrac{\partial h_y}{\partial x} - \cancel{\dfrac{\partial h_x}{\partial y}} - j_z\right) \end{cases}$$

Figure 2: Simplification of Maxwell equations in 2D case (no variation along y)

They describe two independent modes of propagation: one is called the Transverse Electric (TE) mode (Fig.2 left side) because the component perpendicular to the incidence plane is E_y. The second mode is called Transverse Magnetic (TM) mode (Fig.2 right side) because the component perpendicular to the incident plane is H_y.

They propagate independently and then can be modelled and acquired separately. They don't' have the same sensitivity to resistive or conductive targets.

A plane wave propagating along the wave vector $\vec{k}$ in TE mode is depicted Fig. 3-left,

IFP Energies Nouvelles ebook: JL Mari et al., Electromagnetic methods in Geophysics, 2016,
DOI: 10.2516/ifpen/2017001
http://books.ifpenergiesnouvelles.fr/ebooks/ifpen-electro/

whereas the TM mode is depicted Fig.3-right.

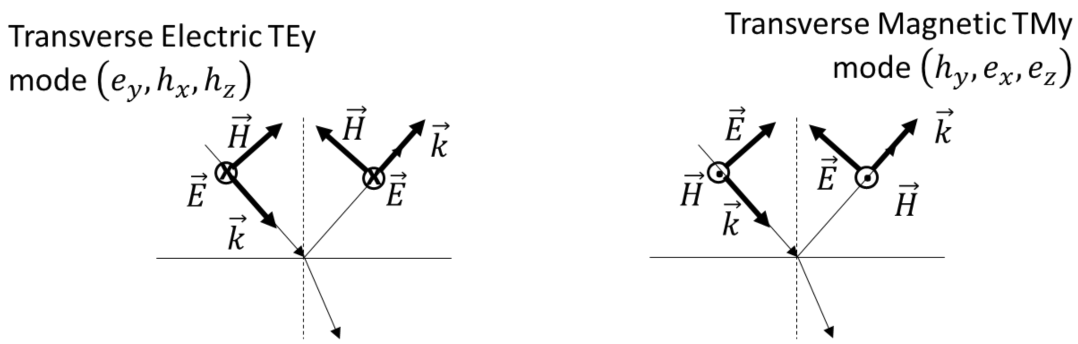

Figure 3. EM plane wave propagating along the wave vector $\vec{k}$, in TE mode (left) and in TM mode (right)

Expressions of Maxwell equations in 1D

If there is no variation along x and y, then we are in a 1D case. Only the derivatives along z are non-zero.

$$\begin{cases} \dfrac{\partial h_x}{\partial t} = \dfrac{1}{\mu}\dfrac{\partial e_y}{\partial z} \\ \dfrac{\partial e_y}{\partial t} = \dfrac{1}{\varepsilon}\left(\dfrac{\partial h_x}{\partial z} - \sigma e_y\right) \\ \dfrac{\partial h_z}{\partial t} = 0 \end{cases} \qquad \begin{cases} \dfrac{\partial e_x}{\partial t} = -\dfrac{1}{\varepsilon}\left(\dfrac{\partial h_y}{\partial z} + \sigma_x\right) \\ \dfrac{\partial h_y}{\partial t} = -\dfrac{1}{\mu}\left(\dfrac{\partial e_x}{\partial z}\right) \\ \dfrac{\partial e_z}{\partial t} = -\dfrac{1}{\varepsilon} j_z \end{cases}$$

Transverse Electric TExy mode

Transverse Magnetic TMxy mode

Figure 4 – Simplified Maxwell equations in 1D, TE mode (left) and TM mode (right).

Half space case

In the general case, for a plane wave propagating along $\vec{k}$, we can write:

$\vec{e} = \vec{e}_0 e^{-i(\vec{k}.\vec{r}-\omega t)}$. In this case, the rotational is: $\vec{\nabla} \times \vec{e} = -i\vec{k} \times \vec{e}$.

Because $\vec{\nabla} \times \vec{e} = -\mu \dfrac{\partial \vec{h}}{\partial t}$ then substituting in $\vec{\nabla} \times \vec{E} = -i\omega\mu\vec{H}$ then $\vec{H} = \frac{1}{\mu\omega}\vec{k} \times \vec{E}$.

We define the electromagnetic impedance as the ratio of the Electric and Magnetic components: in the 1D case considered above: $Z = \frac{E_x}{H_y} = \frac{\mu\omega}{k}$.

We define two independent modes, one is carried by E_y, H_x called TE_{xy} (Fig.4 left) and the other one is carried by E_x,H_y called TM_{xy} (Fig.4 right).

If we consider the 1D TE_{xy} case, $h_x$and e_y are solutions of the same Helmholtz equation:

$$\frac{\partial^2 e_y}{\partial t^2} = \frac{1}{\mu\varepsilon}\left(\frac{\partial^2 e_y}{\partial z^2} - \mu\sigma\frac{\partial e_y}{\partial t}\right)$$

A solution in homogeneous and isotropic medium is a plane wave, described by:

$$\overrightarrow{e_y} = \overrightarrow{e_{y0}^{+}} e^{-i(kz-\omega t)} + \overrightarrow{e_{y0}^{-}} e^{+i(kz-\omega t)}$$

In the quasi-static assumption, $k = \sqrt{i\mu\omega\sigma}$ then $Z = \sqrt{2\pi f \rho\mu_0} e^{-i\pi/4}$.

Where we derive the Cagniard formula, in 1D:

$$\begin{cases} \rho_{app} = \dfrac{1}{2\pi\mu_0 f}|Z|^2 \\ \varphi = -\pi/4 \end{cases}$$

This is valid only in 1D and if we are in the plane wave assumption. This last condition happens when we are in the far field domain.

Figure 5 and 6 depict the numerical computation of Z=E/H at air – ground interface, for a 2 kHz frequency above a homogeneous 500 Ohm.m resistivity ground. The ratio Z is varying near the transmitter and reaches a constant value when the distance is above one wavelength.

This is an illustration of the transition from near to far field.

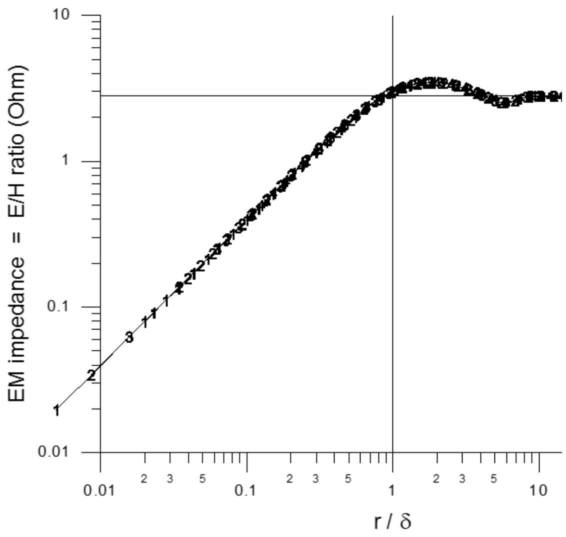

Figure 5 - Electromagnetic impedance E/H for a Vertical Magnetic Dipole source VMD (courtesy of B. Bourgeois, BRGM).

Whatever the nature of the transmitter, magnetic or electric dipole (Fig. 5 & 6), beyond a distance $\lambda = 2\pi\delta$ the impedance reaches a constant value controlled by the previous Cagniard formula.

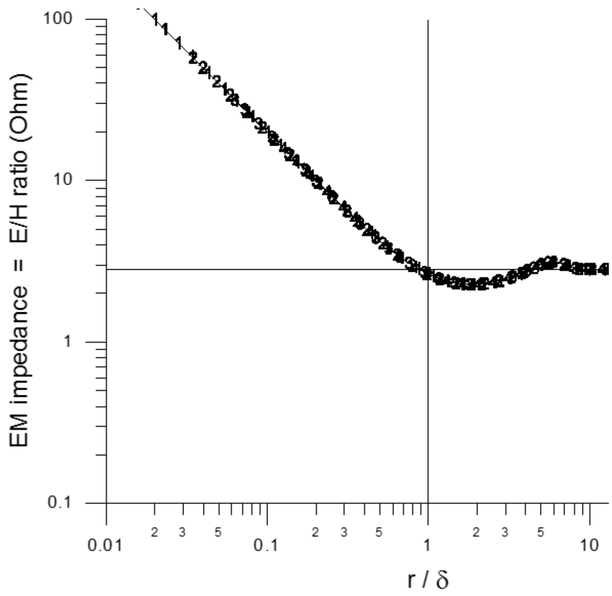

Figure 6 - Electromagnetic impedance E/H for a Vertical Electric Dipole source (courtesy of B. Bourgeois, BRGM).

In far field, Z is constant and depends only on the frequency and resistivity of the ground.

Far field behaviour, and frequency sounding effect

Due to the air ground resistivity contrast, whatever the incidence at the air-ground interface, the EM wave propagates at normal incidence in the formation.

In the far field assumption and in 1D, as E and H are perpendicular to the wave vector $\vec{k}$, then the electric and magnetic current lines are horizontal (see Fig. 7, upper right side of the dotted line).

The depth of investigation depend on the skin depth $\delta = 503\sqrt{\rho/f}$ (attenuation of 1/e every δ). One can see the polarity changes every $\lambda/2$.

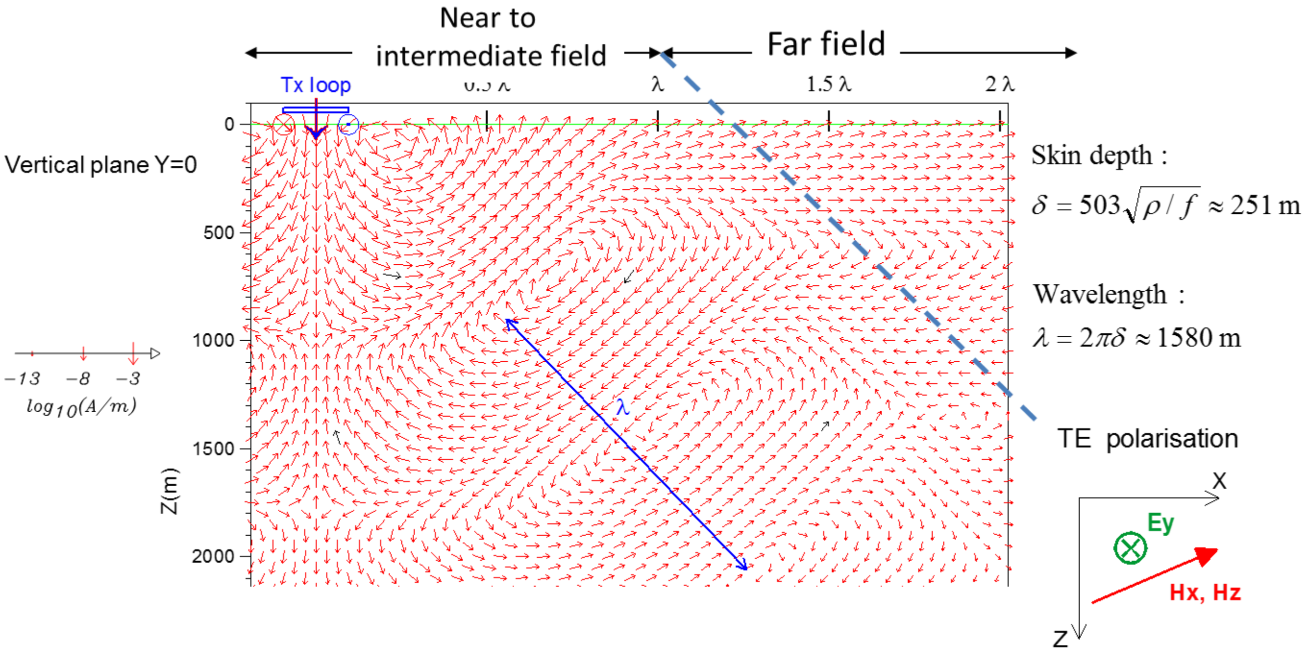

Figure 7 Magnetic field (quadrature component) created by a 300 m square loop (≈VMD), in a 500 Ω.m half-space at 2kHz (courtesy of B. Bourgeois, BRGM)

Magneto-Telluric impedance tensor

Practically, we measure the components of E and H. We estimate Z the MT impedance tensor defined as :

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix}$$

By analogy with the 1D case, one can define $\rho_{ij}(\omega) = \frac{1}{\omega\mu}\left|Z_{ij}(\omega)\right|^2$

and the phase:

$$\varphi_{ij}(\omega) = \arg\left(Z_{ij}(\omega)\right) = \operatorname{atan}\left(\frac{Im\left(Z_{ij}(\omega)\right)}{Re\left(Z_{ij}(\omega)\right)}\right)$$

- If the estimation of $Z_{xx} \approx Z_{yy} \approx 0$ and $Z_{xy} \approx -Z_{yx}$ the impedance tensor is reduced to:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 & Z \\ -Z & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix}$$

Then we are in the 1D case, and the Cagniard formula can be directly used.

- If the estimation of $Z_{xx} \approx Z_{yy} \approx 0$ and $Z_{xy} \neq -Z_{yx}$ the impedance tensor is reduced to:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix}$$

Then we are in the 2D case, easier to handle than 3D.

If x is along the profile and y perpendicular, the incidence plane is the along the measured profile and the impedance tensor provides the TE and TM components:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 & Z_{TM} \\ Z_{TE} & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix}$$

Modelling and inversion can be performed in 2D, and only need computing 3 components in each case (TE or TM).

Dimensionality indicator

If the signal to noise ratio is good, one can define a dimensionality indicator based on the determinant of Z. For instance, the Swift skew (Swift, 1967) can help to determine if the data can be considered 2D (here if $\kappa < 0.3$):

$$\kappa = \frac{Z_{xx} + Z_{yy}}{Z_{xy} - Z_{yx}}$$

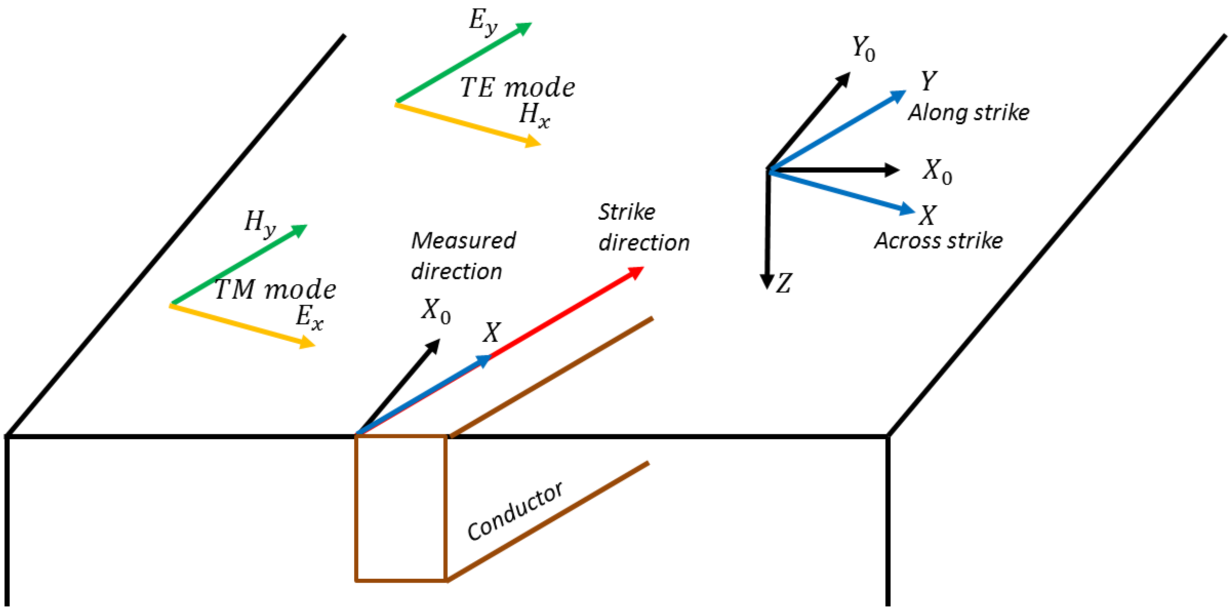

Figure 8 Scheme of a 2D ground (conductor elongated along Y and perpendicular to X direction) and EM field components measured in surface along X_0,Y_0 directions.

Rotation of measured data

In a 2D earth, a conductor is oriented along the "geological strike" direction. But, if measured E_0 & H_0 components are not parallel and perpendicular to the geological strike, they can be rotated in order to obtain the TE and TM modes where E_{rot} (or respectively H_{rot}) is parallel to the strike using a rotation matrix R:

$Z_{rot} = RZR^T$ where $R = \begin{bmatrix} cos\varphi & sin\varphi \\ -sin\varphi & cos\varphi \end{bmatrix}$

Looking for the best angle φ (i.e. the geological strike direction) is part of the preprocessing of the dataset.

Use of the vertical magnetic component

One can define the Tipper, also called « Induction Vector», which relates the vertical magnetic component to the horizontal components.

$$H_z = [T_{zx} \quad T_{zy}] \begin{bmatrix} H_x \\ H_y \end{bmatrix}$$

This vector can be mapped, then it indicates the conductors which induced this vertical magnetic component and its amplitude is link to the size and conductivity of this body.

Acknowledgements

We thank IFPEN for publishing that article destined to help beginners to tackle MT method.

References

Bourgeois B., Suignard K., Perrusson G., *2000. Electric and magnetic dipoles for geometric interpretation of three-component electromagnetic data in geophysics*. Inverse Problems, 16, p1225-1261.

Cagniard L. (1953). Basic theory of the magnetotelluric method of geophysical prospecting. Geophysics, 18(3), 605-635.

Chave A.D. (Editor) and Jones A. (Editor) (2012). The magnetotelluric method: theory and practice. Cambridge University Press, 548 pp.

Chave A.D. and Thomson D.J. (2004). Bounded influence magnetotelluric response function estimation. Geophysical Journal International, 157(3), 988-1006.

Tournerie B., Chouteau M., Marcotte D. (2007). Magnetotelluric static shift: Estimation and removal using the cokriging method. Geophysics, 72(1), F25-F34.

Vozoff K. (1991). The magnetotelluric method. In: M.N. Nabighian (Editor), Electromagnetic Methods in Geophysics. Society of Exploration Geophysicists, Tulsa, Oklahoma, USA, pp. 641-711.

Wawrzyniak P., Sailhac P. and Marquis G. (2013) – Robust error on magnetotelluric impedance estimates. Geophysical Prospecting, 61, 533-546, doi: 10.1111/j.1365-2478.2012.01094.x.